D MULTIPLICATIONS INVOLVING RADICALS

The rules for expanding expressions containing radicals are identical to those for ordinary algebra.

$$\begin{aligned} a(b+c) &= ab+ac \\ (a+b)(c+d) &= ac+ad+bc+bd \\ (a+b)^2 &= a^2+2ab+b^2 \\ (a+b)(a-b) &= a^2-b^2 \end{aligned}$$

Example 8 **Self Tutor**

Expand and simplify:

a $\sqrt{2}(\sqrt{2}+\sqrt{3})$

b $\sqrt{3}(6-2\sqrt{3})$

Example 8 **Self Tutor**

Expand and simplify:

a $\sqrt{2}(\sqrt{2}+\sqrt{3})$ **b** $\sqrt{3}(6-2\sqrt{3})$

a
$$\begin{aligned}&\sqrt{2}(\sqrt{2}+\sqrt{3})\\&=\sqrt{2}\times\sqrt{2}+\sqrt{2}\times\sqrt{3}\\&=2+\sqrt{6}\end{aligned}$$

b
$$\begin{aligned}&\sqrt{3}(6-2\sqrt{3})\\&=\sqrt{3}\times 6+\sqrt{3}\times(-2\sqrt{3})\\&=6\sqrt{3}-6\end{aligned}$$

With practice you should not need all of the steps.

Expand and simplify:

a $\sqrt{2}(\sqrt{5}+\sqrt{2})$

b $\sqrt{2}(3-\sqrt{2})$

c $\sqrt{3}(\sqrt{3}+1)$

d $\sqrt{3}(1-\sqrt{3})$

e $\sqrt{7}(7-\sqrt{7})$

f $\sqrt{5}(2-\sqrt{5})$

g $\sqrt{11}(2\sqrt{11}-1)$

h $\sqrt{6}(1-2\sqrt{6})$

i $\sqrt{3}(\sqrt{3}+\sqrt{2}-1)$

j $2\sqrt{3}(\sqrt{3}-\sqrt{5})$

k $2\sqrt{5}(3-\sqrt{5})$

l $3\sqrt{5}(2\sqrt{5}+\sqrt{2})$

Example 9

Self Tutor

Expand and simplify:

a $-\sqrt{2}(\sqrt{2}+3)$

b $-\sqrt{3}(7-2\sqrt{3})$

Example 9

Self Tutor

Expand and simplify:

a $-\sqrt{2}(\sqrt{2}+3)$

b $-\sqrt{3}(7-2\sqrt{3})$

a
$$\begin{aligned}&-\sqrt{2}(\sqrt{2}+3)\\ &= -\sqrt{2}\times\sqrt{2} + -\sqrt{2}\times 3\\ &= -2-3\sqrt{2}\end{aligned}$$

b
$$\begin{aligned}&-\sqrt{3}(7-2\sqrt{3})\\ &= -\sqrt{3}\times 7 + -\sqrt{3}\times(-2\sqrt{3})\\ &= -7\sqrt{3}+6\end{aligned}$$

2 Expand and simplify:

a $-\sqrt{2}(3-\sqrt{2})$
b $-\sqrt{2}(\sqrt{2}+\sqrt{3})$
c $-\sqrt{2}(4-\sqrt{2})$
d $-\sqrt{3}(1+\sqrt{3})$
e $-\sqrt{3}(\sqrt{3}+2)$
f $-\sqrt{5}(2+\sqrt{5})$
g $-(\sqrt{2}+3)$
h $-\sqrt{5}(\sqrt{5}-4)$
i $-(3-\sqrt{7})$
j $-\sqrt{11}(2-\sqrt{11})$
k $-(\sqrt{3}-\sqrt{7})$
l $-2\sqrt{2}(1-\sqrt{2})$
m $-3\sqrt{3}(5-\sqrt{3})$
n $-7\sqrt{2}(\sqrt{2}+\sqrt{6})$
o $(-\sqrt{2})^3(3-\sqrt{2})$

Example 10

Self Tutor

Expand and simplify: $(3-\sqrt{2})(4+2\sqrt{2})$

Example 10

Self Tutor

Expand and simplify: $(3-\sqrt{2})(4+2\sqrt{2})$

$$\begin{aligned}
&(3-\sqrt{2})(4+2\sqrt{2}) \\
&= 12 + 3\times 2\sqrt{2} + (-\sqrt{2})\times 4 + (-\sqrt{2})\times 2\sqrt{2} \quad \{\text{FOIL rule}\} \\
&= 12 + 6\sqrt{2} - 4\sqrt{2} - 4 \\
&= 8 + 2\sqrt{2}
\end{aligned}$$

3 Expand and simplify:

a $(1+\sqrt{2})(2+\sqrt{2})$

b $(2+\sqrt{3})(2+\sqrt{3})$

c $(\sqrt{3}+2)(\sqrt{3}-1)$

d $(4-\sqrt{2})(3+\sqrt{2})$

e $(1+\sqrt{3})(1-\sqrt{3})$

f $(5+\sqrt{7})(2-\sqrt{7})$

g $(\sqrt{5}+2)(\sqrt{5}-3)$

h $(2\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

i $(4-\sqrt{2})(3-\sqrt{2})$

Example 11

Self Tutor

Expand and simplify:

a $(\sqrt{3}+2)^2$

b $(\sqrt{3}-\sqrt{7})^2$

Example 11

Self Tutor

Expand and simplify:

a $(\sqrt{3}+2)^2$

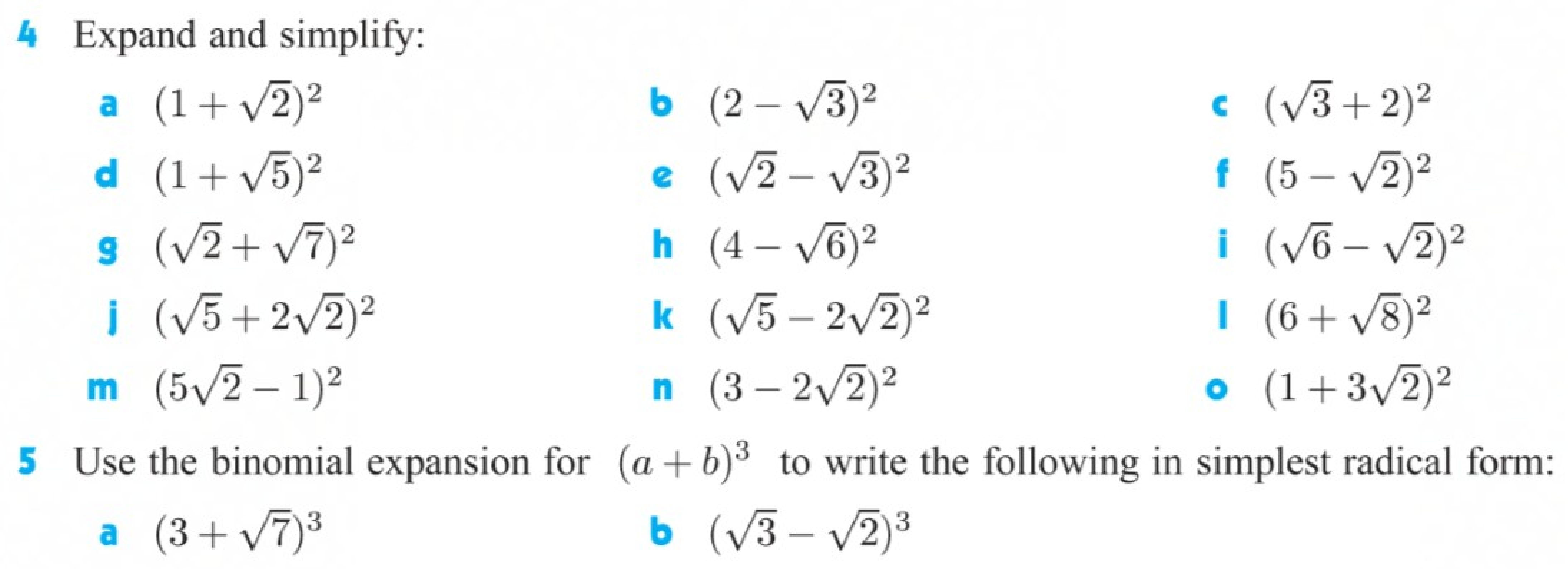
b $(\sqrt{3}-\sqrt{7})^2$

a

$$\begin{aligned}
&(\sqrt{3}+2)^2 \\
&= (\sqrt{3})^2 \ + \ 2 \times \sqrt{3} \times 2 \ + \ 2^2 \\
&= 3 + 4\sqrt{3} + 4 \\
&= 7 + 4\sqrt{3}
\end{aligned}$$

b

$$\begin{aligned}
&(\sqrt{3}-\sqrt{7})^2 \\
&= (\sqrt{3})^2 \ + \ 2 \times \sqrt{3} \times (-\sqrt{7}) \ + \ (-\sqrt{7})^2 \\
&= 3 - 2\sqrt{21} + 7 \\
&= 10 - 2\sqrt{21}
\end{aligned}$$



4 Expand and simplify:

- **a** $(1+\sqrt{2})^2$
- **b** $(2-\sqrt{3})^2$
- **c** $(\sqrt{3}+2)^2$
- **d** $(1+\sqrt{5})^2$
- **e** $(\sqrt{2}-\sqrt{3})^2$
- **f** $(5-\sqrt{2})^2$
- **g** $(\sqrt{2}+\sqrt{7})^2$
- **h** $(4-\sqrt{6})^2$
- **i** $(\sqrt{6}-\sqrt{2})^2$
- **j** $(\sqrt{5}+2\sqrt{2})^2$
- **k** $(\sqrt{5}-2\sqrt{2})^2$
- **l** $(6+\sqrt{8})^2$
- **m** $(5\sqrt{2}-1)^2$
- **n** $(3-2\sqrt{2})^2$
- **o** $(1+3\sqrt{2})^2$

5 Use the binomial expansion for $(a+b)^3$ to write the following in simplest radical form:

- **a** $(3+\sqrt{7})^3$
- **b** $(\sqrt{3}-\sqrt{2})^3$

Example 12

Self Tutor

Expand and simplify:

a $(3+\sqrt{2})(3-\sqrt{2})$

b $(2\sqrt{3}-5)(2\sqrt{3}+5)$

Example 12 **Self Tutor**

Expand and simplify:

a $(3+\sqrt{2})(3-\sqrt{2})$ **b** $(2\sqrt{3}-5)(2\sqrt{3}+5)$

a $(3+\sqrt{2})(3-\sqrt{2})$

$= 3^2 - (\sqrt{2})^2$

$= 9 - 2$

$= 7$

b $(2\sqrt{3}-5)(2\sqrt{3}+5)$

$= (2\sqrt{3})^2 - 5^2$

$= (4 \times 3) - 25$

$= 12 - 25$

$= -13$

Did you notice that these answers are **integers**?

6 Expand and simplify:

a $(4+\sqrt{3})(4-\sqrt{3})$
b $(5-\sqrt{2})(5+\sqrt{2})$
c $(\sqrt{5}-2)(\sqrt{5}+2)$
d $(\sqrt{7}+4)(\sqrt{7}-4)$
e $(3\sqrt{2}+2)(3\sqrt{2}-2)$
f $(2\sqrt{5}-1)(2\sqrt{5}+1)$
g $(5-3\sqrt{3})(5+3\sqrt{3})$
h $(2-4\sqrt{2})(2+4\sqrt{2})$
i $(1+5\sqrt{7})(1-5\sqrt{7})$

7 Expand and simplify:

a $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$
b $(\sqrt{7}+\sqrt{11})(\sqrt{7}-\sqrt{11})$
c $(\sqrt{x}-\sqrt{y})(\sqrt{y}+\sqrt{x})$
d $(3\sqrt{2}+\sqrt{3})(3\sqrt{2}-\sqrt{3})$
e $(3\sqrt{3}+\sqrt{7})(3\sqrt{3}-\sqrt{7})$
f $(2\sqrt{5}-3\sqrt{2})(3\sqrt{2}+2\sqrt{5})$

1 **a** $\sqrt{10}+2$ **b** $3\sqrt{2}-2$ **c** $3+\sqrt{3}$
d $\sqrt{3}-3$ **e** $7\sqrt{7}-7$ **f** $2\sqrt{5}-5$
g $22-\sqrt{11}$ **h** $\sqrt{6}-12$ **i** $3+\sqrt{6}-\sqrt{3}$
j $6-2\sqrt{15}$ **k** $6\sqrt{5}-10$ **l** $30+3\sqrt{10}$

2 **a** $2-3\sqrt{2}$ **b** $-2-\sqrt{6}$ **c** $2-4\sqrt{2}$
d $-3-\sqrt{3}$ **e** $-3-2\sqrt{3}$ **f** $-5-2\sqrt{5}$
g $-3-\sqrt{2}$ **h** $-5+4\sqrt{5}$ **i** $\sqrt{7}-3$
j $11-2\sqrt{11}$ **k** $\sqrt{7}-\sqrt{3}$ **l** $4-2\sqrt{2}$
m $9-15\sqrt{3}$ **n** $-14-14\sqrt{3}$ **o** $4-6\sqrt{2}$

3 **a** $4+3\sqrt{2}$ **b** $7+4\sqrt{3}$ **c** $1+\sqrt{3}$
d $10+\sqrt{2}$ **e** -2 **f** $3-3\sqrt{7}$
g $-1-\sqrt{5}$ **h** $1-\sqrt{6}$ **i** $14-7\sqrt{2}$

4 **a** $3+2\sqrt{2}$ **b** $7-4\sqrt{3}$ **c** $7+4\sqrt{3}$
d $6+2\sqrt{5}$ **e** $5-2\sqrt{6}$ **f** $27-10\sqrt{2}$
g $9+2\sqrt{14}$ **h** $22-8\sqrt{6}$ **i** $8-4\sqrt{3}$
j $13+4\sqrt{10}$ **k** $13-4\sqrt{10}$ **l** $44+24\sqrt{2}$
m $51-10\sqrt{2}$ **n** $17-12\sqrt{2}$ **o** $19+6\sqrt{2}$

5 **a** $90+34\sqrt{7}$ **b** $9\sqrt{3}-11\sqrt{2}$

6 **a** 13 **b** 23 **c** 1 **d** -9 **e** 14
f 19 **g** -2 **h** -28 **i** -174

7 **a** 1 **b** -4 **c** $x-y$ **d** 15 **e** 20 **f** 2